Name ______________________________ **Final Exam Review**

Instructor: Mircea Martin **MATH 142**

Please **read** carefully, **show** all your work, and **explain** all your answers.
Credit may be reduced if work is incomplete.

Problem 1. Categorize these as either expressions or sentences.

a) $x + 3 = 5$

b) $x + 3$

c) $x + 3 < 5$

d) $x + 2x = 3x$

Problem 2.

a) Thinking of Mathematics as a language, what parts of speech are expressions?

b) Thinking of Mathematics as a language, what part of speech is "="?

Problem 3. $1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 3 = 3$, $1 \times 4 = 4$, $1 \times 5 = 5$, $1 \times 6 = 6$, ...
State, symbolically, the corresponding abstract fact about multiplication by 1.

Problem 4.

a) What is the term for an equation that is always true, regardless of the value of its variables?

b) What is the term for a sentence with a variable that asserts that something is always true?

Problem 5.

a) Define *equivalent equations.*

b) Give two equations that are equivalent.

c) Give two equations that are not equivalent.

Problem 6. Which are dummy variables and which are unknowns?

a) $x + 3 = 5$

b) $x + 3 = 3 + x$

c) $x < x + 3$

d) $2x < x + 3$

Problem 7.

a) The expression $2x + 3$ simultaneously represents two distinct types of mathematical objects. Which two *types* of objects does it represent?

b) What is the difference between an identity and an equation?

Problem 8. Evaluate.

a) $48/(6 \cdot 2)$

b) $\dfrac{8(-2)+6}{6-1}$.

Problem 9. Name the property illustrated by the sentence

a) $2(3+4) = 2 \cdot 3 + 2 \cdot 4$

b) $2 + (3+4) = (2+3) + 4$

c) $2(2+3) = 2(4+3)$

d) $1(3+4) = 3+4$

Problem 10. Write out, in English, the proper pronunciation of the following sentences.

a) $3x^3 + \sqrt{y} \geq z$

b) $\dfrac{-2+3x}{4} = 5$

Problem 11. Let $a = 1$, $b = -2$, and $c = 3$. Evaluate

a) $\dfrac{a^2+b}{c} =$

b) $a + (b-c)^2 =$

Problem 12.

a) Solve the inequality $2x - 5 \geq -x + 4$. Express your answer as $x \geq a$.

b) Solve the inequality $|x - 7| < 5$. Express your answer as $a < x < b$.

Problem 13.

a) $\{x \mid 2 \leq x < 3\}$. Write out, in English, how you would say these aloud.

b) $4 \in \{x \mid x \geq 1\}$. Give the defining property (i.e., set builder notation) for $(-3, 7]$.

Problem 14. Given $S = [-3, 6)$ and $T = (4, 12]$, find

a) $S \cap T =$

b) $S \cup T =$

c) $S^c \cap T =$

d) $S \cap T^c =$

Problem 15. Given $S = \{1, 2, 3, 4, 5, 6\}$, $T = \{4, 5, 6, 7, 8\}$, and the universal set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, find

$S \cap T =$

$(S \cap T)^{\mathrm{c}} =$

S^{c} and T^{c}; $S^{\mathrm{c}} =$, $T^{\mathrm{c}} =$

$S^{\mathrm{c}} \cup T^{\mathrm{c}} =$

Problem 16. Give each of the following functions as a command.

a) $f(x) = 4x^3 - 5$

b) $f(x) = \sqrt{x - 2}$

Problem 17. Give the "$f(x)$" definitions of the functions

a) Add 3 and then take the absolute value.

b) Square, add 5, and then divide by 2.

Problem 18. Given $f(x) = x^3$ and $g(x) = x - 2$, find

a) $g(f(x)) =$

b) $f(g(x)) =$

Problem 19. Find two simpler functions f and g such that $f(g(x)) = \sqrt{x^3 + 2}$.

Problem 20. The cost of photocopies is 5 cents each for the first 8, and 3 cents each for every copy after the first 8.

a) Express the function that gives the cost of x copies.

b) What is the domain of the function in part a)?

Problem 21. Which of the following processes applied to both sides of an equation always produce an *equivalent* equation? Circle your choice(s).

a) add 7
b) multiply by 3
c) cancel a common factor of $x^2 + 1$
d) divide by 6
e) square
f) multiply by x

Problem 22. Solve using the inverse-reverse method after adapting the equation to fit the format requirement.

a) $3x = 20 - 2x$

b) $x^2 = 55 - 2x^2$

c) $x + 4\pi x = 20$

d) $-9 = -2x + 21$

Problem 23. Determine if the step from *E1* to *E2* is justified by a rule from "Rules of Algebra." If not, say so and also name the rule and give the connective.

a) *E1*: $(x+2)f(x) = 3(x+2)$ *E2*: $f(x) = 3$

b) *E1*: $x^2 = 4x - 7$ *E2*: $x^2 - 4x + 7 = 0$

Problem 24. Let *E1* and *E2* be two equations such that *E1* $\Rightarrow$ *E2*.

a) How do their solution sets compare? Can they be equal?

b) If one has more solutions than the other, which is it?

Problem 25. Solve step by step. Exhibit every step, exhibit the connective, and cite one of Rules 1–9 from "Rules of Algebra", for each step.

a) $x + 1 = \sqrt{13 - 2x}$

b) $(x^2 + 3x)/(x - 1) = 4/(x - 1)$

Problem 26. A triangle has three sides. One is twice as long as the second, and the third is 12 centimeters long. The perimeter is 33 centimeters.

a) Use "x" to represent the length of the shorter unknown side anf build a formula for the perimeter.

b) Set up the relevant equation.

c) Solve for x.

Problem 27.

a) A triangle has base 8 and area 78. What is its perimeter?

b) A triangle has perimeter 26 and area 30. What is its base?

c) Earl could rent a car A for 30$plus35centspermile.Hecouldrentcar$B$for$40 plus 26 cents per mile. How many miles would he have to drive to make car B a better deal?

Problem 28. Construct truth tables for:

a) (not B) $\Rightarrow$ (not A)

b) $(A \wedge B)$ or [(not A) $\wedge$ (nor B)]

Problem 29. Prove that the statement "$H \wedge (H \Rightarrow C) \Rightarrow C$" is always true. This fact is known as *Modus Ponens*.

Problem 30. State the contrapositive of these:

a) If $x > 7$, then $|x| > 7$.

b) If the chair is an antique, it is over 75 years old.

Problem 31. Assuming that A is a **true (T)** statement and B is **false (F)**, indicate the truth value of each of the following compound statements.

a) (not$A$) or B

b) A and (not$B$)

c) $A \Longrightarrow B$

d) $B \Longrightarrow A$

e) $A \Longleftrightarrow (A \text{ or } B)$

Problem 32. There are five connectives. Name three of them and for each give the corresponding set theory concept.

Problem 33. *True or false?* (No reason required.)

a) "A and B" is logically equivalent to "B and A"

b) "A or B" is logically equivalent to "B or A"

c) "A $\Longrightarrow$ B" is logically equivalent to "B $\Longrightarrow$ A"

d) "A $\Longleftrightarrow$ B" is logically equivalent to "B $\Longleftrightarrow$ A"

Problem 34.

a) Restate, using the *theorem on cases*:
"If your earned income was more than $3700 or your unearned income was more than $1300, you must file a return."

b) Restate, using the *theorem on a hypothesis in the conclusion*:
"If you attend BU, if you win the math contest, you get a trip to New York."

c) Restate using a *version of the contrapositive*:
"When I am cold and wet, I am miserable."

d) Restate using the *theorem on "or" in the conclusion*:
"When I go on vacation, I go to the ocean or to the mountains."

Problem 35. *True or false?* (No reason required.)

a) The contrapositive of a conditional is logically equivalent to the original conditional.

b) The converse of a conditional is logically equivalent to the original conditional.

c) The contrapositive of the contrapositive of a conditional is logically equivalent to the original conditional.

d) The converse of the converse of a conditional is logically equivalent to the original conditional.

Problem 36. Use *DeMorgan's laws.* If this is false, what is true?

a) "She is tall or smart."

b) "He is taking math and physics."

Problem 37. Give the definition and an example of a

a) *Contradiction*

b) *Tautology*

Problem 38.

a) Give a simple form of the negation of $A \Rightarrow (A \text{ or } B)$

b) Give a simple form of the contrapositive of $A \Rightarrow (A \text{ or } B)$

Problem 39.

a) True or false? (A and B) $\Rightarrow$ (A or B).

b) State the set-theory result corresponding to (A and B) $\Rightarrow$ (A or B).

Problem 40. Suppose "TV sitcoms in prime time make lots of money," is true. What can be deduced from the given additional facts?

a) "The TV show is a sitcom in prime time."

b) "The TV show makes lots of money."

c) "The TV show does not make lots of money."

d) "The TV show is a sitcom and does not make lots of money."

Problem 41. Decide if you are seeing a *true generalization* or an *open sentence.* For each part, decide if the variables are free variables or dummy variables.

a) $3(x + 2) = 3x + 6$.

b) $6x = 12$.

c) $[A \text{ and } (\text{not } A)] \Rightarrow B$

d) $A \Rightarrow [B \text{ or } (\text{not } B)]$.

e) $S = T^c$.

f) $S \cap S^c = \emptyset$.

Problem 42.

a) Suppose "h" is a known function and we intend to define "f" by the following sentences which are intended to hold "for all x". Which fit the convention for a *definition* of "f"?

A. $h(x) + 3 = f(x)$.

B. $f(x) = [h(x)]^3$.

C. $f(x) = x^3$.

D. $x^3 = f(x)$.

b) Suppose "T" is a known set and we intend to define the set "S" by the following sentences. Which fit the convention for a *definition* of "S"?

A. $S = T^c$.

B. $T^c = S$.

C. $S = T \cup [0, 2]$.

D. $(0, 2) \cup S = T$.

Problem 43. If the given sentence were preceded by the appropriate "For all. . . ", which variables would be mentioned there?

a) $S \subset U$.

b) $S \cap T \subset S$.

c) $a = b$ iff $a - c = b - c$.

Problem 44. Suppose the following assertions appear in the middle of a mathematical paragraph. Decide if you are seeing a *true generalization* (definition, identity, tautology) or an *open sentence* which depends upon the meaning of the letters employed (equation, statement from logic).

a) $ab = ba$

b) $a - b = a - c$

c) Suppose $f(x) = x^2$

d) $A \wedge (B \Rightarrow C) \Rightarrow C$.

e) $c^2 = c + 3$.

f) $c(c + 3) = c^2 + 3c$.

Problem 45.

a) What is the variable?

A. $|x| < 2$ iff $-2 < x < 2$.

B. $x < y \Rightarrow x^3 < y^3$.

C. $a = b$ iff $a - c = b - c$.

D. $a(b - c) = ab - ac$.

b) Consider the sentence "$y = ax^2 + bx + c$."

A. How are the various letters to be interpreted?

B. How do you know how to interpret it?

C. Are you supposed to solve it?

Problem 46.

a) What type of statement is the double negation of a generalization?

b) What type of statement is the negation of an existence statement?

Problem 47. Give the negation, in positive form, of

a) "For all y, $h(y) \leq 10$."

b) "For all x in T, $x \geq 3$."

Problem 48.

a) Restate as a conditional sentence: "Sets containing S also contain R."

b) Restate as an equivalent conditional sentence without variables: "The product of four consecutive integers is divisible by twentyfour."

c) Restate it with variables.

Problem 49.

a) Restate as a conditional sentence: "Rectangles with perpendicular diagonals are squares."

b) State its contrapositive.

Problem 50. Give a simplified version of the contrapositive of "$A \Rightarrow (B \text{ and } C)$."