Name: _____ **MATH 142, Test 3**

Instructor: Mircea Martin SAMPLE

Please **read** carefully, **show** all your work, and **explain** all your answers.
Credit may be reduced if work is incomplete.

Note: Each problem is worth 10 points.

Problem 1. Give a simple form of the negation of

A. $A \Rightarrow (\text{not } B)$

B. $(\text{not } A) \Rightarrow B$

Problem 2. Give a simple form of the contrapositive of

A. $A \Rightarrow (\text{not } B)$

B. $(\text{not } A) \Rightarrow B$

Problem 3. True or false? $(A \text{ and } B) \Rightarrow A$.

Problem 4. State the set-theory result corresponding to $(A \text{ and } B) \Rightarrow A$.

Problem 5. Suppose "Pro basketball players are tall or quick," is true. What can be deduced from the given additional facts?

A. "John is tall."

B. "John is a pro basketball player and not tall."

C. "John is not tall."

D. "John is not tall or not quick."

E. "John is not tall and not quick."

F. "John is a pro basketball player and tall."

Problem 6. Suppose "TV sitcoms in prime time make lots of money," is true. What can be deduced from the given additional facts?

A. "The TV show is a sitcom."

B. "The TV show makes lots of money."

C. "The TV show does not make lots of money."

D. "The TV show is in prime time and does not make lots of money."

Problem 7. Decide if you are seeing a *true generalization* or an *open sentence*. For each part, decide if the variables are free variables or dummy variables.

A. $3(x+5) = 3x+15$.

B. $3x = 12$.

C. A or (not A)

D. $A \Rightarrow B$.

E. $S^c = T$.

F. $S \cup S^c = U$.

Problem 8. Suppose "h" is a known function and we intend to define "f" by the following sentences which are intended to hold "for all x". Which fit the convention for a *definition* of "f"?

A. $h(x) + 7 = f(x)$.

B. $f(x) = [h(x)]^2$.

C. $f(x) = x^2$.

D. $x^2 = f(x)$.

Problem 9. Suppose "T" is a known set and we intend to define the set "S" by the following sentences. Which fit the convention for a *definition* of "S"?

A. $S = T^c$.

B. $T^c = S$.

C. $S = T \cup [0,1]$.

D. $(0,1) \cup S = T$.

Problem 10. If the given sentence were preceded by the appropriate "For all...", which variables would be mentioned there?

A. $\emptyset \subset S$.

B. $S \subset S \cup T$.

C. $a = b$ iff $a + c = b + c$.

Problem 11. Identify each as an *open sentence, (abbreviated) generalization*, or an *existence statement*. Also identify which type of variable is used in each sentence.

A. If $a \neq 0$ and $ax + b = c$, then $x = (c-b)/a$.

B. $(x+3)(x+2) = x^2 + 5x + 6$.

C. $x^2 = 4x$ has a solution.

D. $5x^2 + 2x = 7x^2$.

E. $S \cap T \subset T$.

F. $R \cap T \subset S$.

Problem 12. Suppose the following assertions appear in the middle of a mathematical paragraph. Decide if you are seeing a *true generalization* (definition, identity, tautology) or an *open sentence* which depends upon the meaning of the letters employed (equation, statement from logic).

A. $a + b = b + a$

B. $a + b = a + c$

C. Suppose $f(x) = 7x$

D. $A \wedge (B \Rightarrow C) \Rightarrow C$.

E. $c^2 = 4c + 7$.

F. $c(c + 1) = c^2 + c$.

Problem 13. What is the variable?

A. $|x| < c$ iff $-c < x < c$.

B. $0 < x < y \Rightarrow x^2 < y^2$.

C. $a = b$ iff $a + c = b + c$.

D. $a(b + c) = ab + ac$.

Problem 14. Consider the sentence "$y = mx + b$."

A. How are the various letters to be interpreted?

B. How do you know how to interpret it?

C. Are you supposed to solve it?

Problem 15.

A. What type of statement is the negation of a generalization?

B. What type of statement is the negation of an existence statement?

Problem 16. Give the negation, in positive form, of

A. "For all y, $h(y) > 10$."

B. "For all x in T, $x \geq 67$."

Problem 17. True or false?

A. There exists $y \in (0, 8)$ such that for all $x \in (0, 8)$, $x \leq y$.

B. For all $x \in (0, 8)$ there exists $y \in (0, 8)$ such that $x \leq y$.

Problem 18. Restate as a conditional sentence: "Sets containing S also contain R."

Problem 19.

A. Restate as an equivalent conditional sentence without variables: "The product of three consecutive integers is divisible by six."

B. Restate it with variables.

Problem 20.

A. Restate as a conditional sentence: "Rectangles with perpendicular diagonals are squares."

B. State its contrapositive.

Problem 21 (EXTRA CREDIT). Give a simplified version of the contrapositive of "$A \Rightarrow (B$ or $C)$."